Calculus Culminating Activity

Part 1 - Optimization Problems

(1) Let x be side length of each square.

$V = L \times W \times H$ where L, W, H are length, width and height respectively. height is x

$(120 - 2x)$ $(50 - 2x)$

$V(x) = (120-2x)(50-2x)x$

$= (120-2x)(50x-2x^2)$

$= 4x^3 - 220x^2 + 600x$

$= x^3 - 55x^2 + 150x$

$x > 0$

$(0, 25)$

$V(x) > 0$

$0 > x < 50$

$V'(x) = x^2 - 55x + 150$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{55 \pm \sqrt{(55)^2 - 4 \times 1 \times 150}}{2 \times 1}$$

$$= \frac{55 \pm \sqrt{2425}}{2}$$

$$= \frac{55 \pm 49}{2}$$

$$\frac{55+49}{2} = 52 \text{ or } 3$$

(3) If the height is h metres and radius the radius is r metres then:

i) cost of the base is $\$90(\pi r^2)$

ii) cost of the top is $\$50(\pi r^2)$

iii) cost of the wall is $\$60(2\pi rh)$

cost of the tank $140\pi r^2 + 120\pi rh$

since $V = \pi r^2 h = 1000$

$$h = \frac{1000}{\pi r^2}$$

$$C(r) = 150\pi r^2 + 160\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$C(r) = 150\pi r^2 + \frac{160000}{r}$$

Part 2 - Sinusoidal Word Problem

a) Maximum height 32m
minimum height 2m

(b) $h = 3t + 2$

(c) $y = 3\sin 3x + 2$

(d) 14 m/s when $t = 4$
86 m/s when $t = 28$

(e) 47m

(f) 72 m/s